E –Learning Material

Skill Enhancement Courses (SEC)

Logic & Sets

Prepared by

<u>Name</u> : Dr. Rima Barik
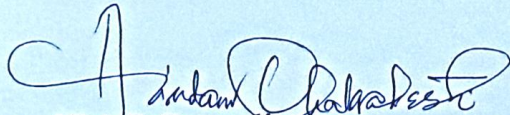
<u>Designation</u>: Assistant Professor

<u>Department</u>: Mathematics

<u>College/University</u> : Khatra Adibasi Mahavidyalaya

Affiliated to Bankura University

<u>Place</u> : Khatra, Bankura

<u>Date</u> : 10/09/2023

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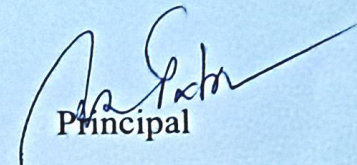<u>*Place*</u> *: Khatra, Bankura*

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IQAC Co-ordinator   Principal

Khatra Adibasi Mahavidyalaya   Khatra Adibasi Mahavidyalaya

Course Title – Logic & Sets

Discipline – Mathematics

Course Level – UG

Semester – 1st

Course Category – Skill Enhancement Course (SEC)

Course Credits – 3

<u>Objective of the module –</u>
  1) To introduce the concept of propositional logic and logical equivalence
  2) To develop the concept predicate logic and quantifiers
  3) To introduce the notion of sets and mappings.

<u>Expected Learning outcome of the module –</u>
 After the completion of this module, the students will be able to
    1) Knowledge : a) define a proposition, tautology, contradiction.
                    b) define predicate and quantifiers
                    c) define mapping and identify different types of mappings

    2) Understanding : d) determine the truth table of any given compound graph
                       e) compare two propositions for their equivalence
                       f) construct the predicate functions

    3) Application: g) execute the problems related to sets, using venn diagram

Syllabus

Unit 1
Introduction, propositions, truth table, negation, conjunction and disjunction. Implications, biconditional propositions, converse, contra positive and inverse propositions and precedence of logical operators. Propositional equivalence: Logical equivalences. Predicates and quantifiers: Introduction, Quantifiers, Binding variables and Negations.

Unit 2
Sets, subsets, Set operations and the laws of set theory and Venn diagrams. Examples of finite and infinite sets. Finite sets and counting principle. Power set of a set. Difference and Symmetric difference of two sets. De Morgan's law, Set identities. Family of sets. Generalized union and intersections. Cartesian product of sets.

Unit 3
Mappings, bijective mappings, composition of mappings, inverse of a mapping.

Quadrant 1 (Video and Audio)

Topics:

1) Introduction to Logic & Sets (part 1)
   https://drive.google.com/file/d/1tjaUHztNOBYlo1VYVew2TIFBLHSPxohN/view

2) Introduction to Logic & Sets (part 2)
   https://drive.google.com/file/d/1haWJTx7AIBTmBsH9nk_aY-5DFkPpNJLZ/view

3) Set Theory
https://drive.google.com/file/d/1q7ZRbLN9IXIdt43MwTCw2qKXTRqpFRle/view

4) Mappings
https://drive.google.com/file/d/17Pq750XUvl23_akyvJO6KdpLv6Q8tvF4/view

Quadrant 2 (e-Text)

- PPTs uploaded in the ERP of the college website.
  https://kamv.ac.in/dept_contents.php?did=14&item_id=7

- OER from University of Mumbai
  Reference link: https://archive.mu.ac.in/myweb_test/S.Y.B.Sc.IT.%20(Sem%20-%20III)%20Logic%20and%20Discrete%20Mathematics.pdf

- OER from University of Houston
  Reference link: https://www.math.uh.edu/~dlabate/settheory_Ashlock.pdf

- **Books :**

1) Set theory and Logic, Robert R. Stoll; Dover Publications, Inc. New York
https://staffnew.uny.ac.id/upload/132319832/pendidikan/ebooksclub.org__Set_Theory_and_Logic.pdf

2) Discrete Mathematical Structures, RM Somasundaram, Prentice Hall of India

Video Link:

1) https://www.youtube.com/watch?v=dtOk0ZHmO4s
2) https://www.youtube.com/watch?v=HkNdNpKUByM

Quadrant 3 (Discussion Forum 30mins)

1) Topic : Logic and Set (Part 1)

LO : identifying tautologies, contradictions and logical equivalences

GA: Knowledge

Pedagogical Approach: Quiz

Time : 20 minutes

Questions: 4

- Composite statements will be given, students have to construct the truth table and identify whether they are tautologies or contradictions or logical equivalences

2) Topic : Logic and Set (Part 2)

LO : Construct the negation of predicates

GA: Understanding

Pedagogical Approach: Quiz

Time : 10 minutes

Questions: 3

- Predicates will be given and students need to formulate the negation of the given predicates.

Objective Type

Q1) Identify which of the followings are not propositions, give reasons in brief   5

        (i) $x + 4 = 10$

        (ii)   Every man has a tail.

        (iii) India is a beautiful country.

        (iv) Open the window.

        (v)   What a surprise !!

        (vi) May I know your name, please ?

Q2) Write the truth value of each of the following compound statements, given that p is T, q and r are both F;   5

    (i) $\neg p \wedge (q \vee r)$ (ii) $(\neg p \wedge q) \vee r)$ (iii) $p \wedge (q \vee r)$ (iv) $\neg (q \wedge r) \wedge p$ (v) $p \wedge q \wedge r$

Q3) Match the items in column A with the items in column B :   5

Column A	**Column B**
(i) In a bijective mapping	a subset of their union
(ii) In a surjective mapping	is zero
(iii) Power set of a set with cardinal number 4	distinct elements have distinct images
(iv) The cardinal number of null set	has 16 elements in it
(v) Symmetric difference of two sets is	the range is equal to the codomain

Q 4) Determine the truth value of the following predicates (Let $\mathbb{Z}$ be the universe of discourse) : 2 X 5 =10

(i) $\forall x \exists y \ (x^2 < y)$ (ii) $\exists x \forall y \ (x < y^2)$ (iii) $\forall x \exists y \ (x + y = 0)$ (iv) $\exists x \exists y \ (x^2 + y^2 = 5)$

(v) $\exists x \exists y (x + y = 4 \ \wedge x - y = 1)$

Short Answer:

Q5) Answer the following questions briefly   3 X 3 = 9

(i) Show that $p \vee (p \rightarrow q)$ is a tautology.
(ii) Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$.
(iii) Show that $A - B \subset C$ if and only if $A - C \subset B$ for any three sets A,B and C.
(iv) Find P(P({0,1})).
(v) If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two surjective mappings, then show that $g_o f : X \rightarrow Z$ is also a surjective mapping.

Long Answer: 5 X 3 = 15

Q6) If $f : X \rightarrow Y$ and $g : Y \rightarrow X$ be two mappings, such that $g_o f = I_X$, then show that f is injective and g is surjective mapping.

Q7) For three sets A, B and C, show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$, but $(A \cup B) \times (C \cup D) \neq (A \times C) \cup (B \times D)$.

Q8) Verify whether $p \rightarrow (q \rightarrow (p \wedge q))$ is a tautology or not.

QUIZ

Topic : Logic and Set (Part 1)

Check whether the followings are tautology/contradiction

Q.1) $((p \rightarrow (q \lor r)) \lor (p \rightarrow q))$

Q.2) $((p \leftrightarrow (\sim q \lor r)) \rightarrow (\sim p \rightarrow q))$

Q.3) $(p \leftrightarrow q) \land (\sim p \land q)$

Q.4) $p \rightarrow (q \rightarrow (p \land q))$

Topic : Logic and Set (Part 2)

Write the negation of the following predicates

Q.1) $\forall x \ (x \leq x^2)$

Q.2) $\forall x \ \exists y \ (x + y = 0)$

Q.3) "All students of Khatra Adibasi Mahavidyalaya are brilliant" (Express using predicate logic)

Q.4) $\forall x \ (x \neq 0) \rightarrow \exists y \ (xy = 1)$.

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QUIZ

Topic : Logic and Set (Part 1)

Check whether the followings are tautology/contradiction

Q.1) $((p \rightarrow (q \lor r)) \lor (p \rightarrow q))$

Q.2) $((p \leftrightarrow (\sim q \lor r)) \rightarrow (\sim p \rightarrow q))$

Q.3) $(p \leftrightarrow q) \land (\sim p \land q)$

Q.4) $p \rightarrow (q \rightarrow (p \land q))$

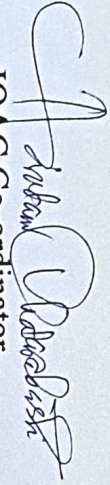## Topic : Logic and Set (Part 2)

Write the negation of the following predicates

Q.1) $\forall x \ (x \leq x^2)$

Q.2) $\forall x \ \exists y \ (x + y = 0)$

Q.3) "All students of Khatra Adibasi Mahavidyalaya are brilliant" (Express using predicate logic)

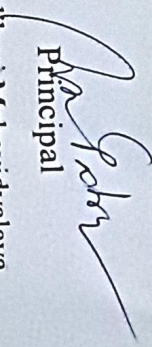Q.4) $\forall x \ (x \neq 0) \rightarrow \exists y \ (xy = 1)$.

IQAC Co-ordinator

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